Technical Knowledge (Continued)

By the end of this class you should be able to:

- Explain several things to examine when working with an equation
- Use a simple Bernoulli equation
- Analyze the units of an equation
- Apply an equation

Notice: Scott Moor's Tuesday office hours changed to 9:30 – 11:30 am

Quantity	Definition or Equation
acceleration	*Change in velocity per time* $= \frac{dv}{dt}$
force	*= ma*
work	*a force acting through a distance = Fd*
energy	*the ability to do work (same units as work)*
power	*(work or energy)/time*
pressure	*force/area*

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

Lumped Parameters: Suppose you can experiment with a given string of fixed length. Derive a lumped parameter model that relates the tension of that string to the frequency using a single parameter that can be determined experimentally.

L, & μ are constant. The equation can be rearranged to:

$$f = \frac{1}{2L\sqrt{\mu}}\sqrt{T} \quad or \quad f = A\sqrt{T}$$

the variable A can be fitted with a simple experiment.

The frequency of a vibrating string (e.g. on a guitar) is given by:

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

where: f = the frequency (s^{-1})
L = the length of the string (m)
T = the tension on the string (units of force, kg m/s^2)
μ = the linear mass of the string (kg/m)

Proportionality: Show two simple proportionalities derived from the above equation.

$$f \propto \frac{1}{L} \qquad f \propto \sqrt{T} \qquad f \propto \frac{1}{\sqrt{\mu}}$$

Symbolic: Mathematical Models Using Equations

Example: The Bernoulli Equation

The Bernoulli Equation

$$\frac{P_1}{\rho}+\frac{1}{2}\left(V_1^{\,2}\right)+gh_1=\frac{P_2}{\rho}+\frac{1}{2}\left(V_2^{\,2}\right)+gh_2$$

Or

$$\frac{\Delta P}{\rho}+\frac{1}{2}\Delta\left(V^2\right)+g\Delta h=0$$

Each term has units of Energy/Mass

➔ a Conservation of Energy Equation

Principles: Conservation Law

<u>What can be conserved?</u>	<u>Examples</u>
• Mass	Mass ➔ continuity in fluid flow
• Energy	Energy ➔ force distance laws (mgl = mgl)
• Momentum (sort of)	Momentum ➔ fluid flow laws
• Charge	Charge ➔ Kirkoff's Current Law
• entropy (sort of)	Entropy ➔ expansion in a valve

What to look for in an Equation

- Dimensions or Units ✓
- Principles ✓
- Assumptions
- Simple relationships
- Simplifications

The Bernoulli Equation: Simple Relationships

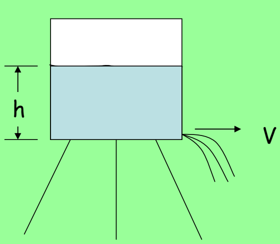
$$\frac{\Delta P}{\rho}+\frac{1}{2}\Delta\left(V^2\right)+g\Delta h=0$$

$\Delta P \propto \Delta V^2$

$\Delta P \propto \Delta h$

$\Delta V^2 \propto \Delta h$

Draining Tank

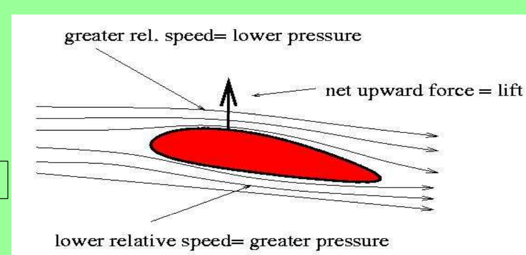


$V^2 \propto h$

Air over a wing

$$\frac{\Delta P}{\rho}+\frac{1}{2}\Delta\left(V^2\right)+\overset{\sim 0}{g\Delta h}=0$$

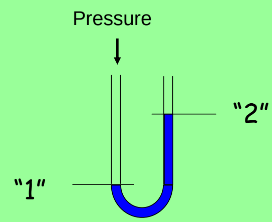
$\boxed{\Delta P \propto -\Delta V^2}$



What to look for in an Equation

- Dimensions or Units ✓
- Principles ✓
- Assumptions
- Simple relationships ✓
- Simplifications

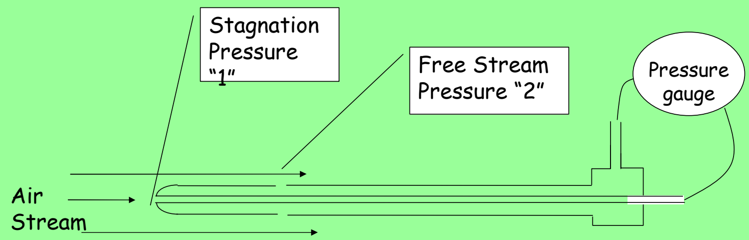
A water manometer



$$\frac{P_1}{\rho} + \frac{1}{2}\cancel{\left(V_1^2\right)} + \cancel{gh_1} = \frac{P_2}{\rho} + \frac{1}{2}\cancel{\left(V_2^2\right)} + gh_2$$

$$\frac{\Delta P}{\rho} = gh_2 \quad or \quad \boxed{\Delta P = \rho g h}$$

A Pitot Tube



$$\frac{P_1}{\rho} + \frac{1}{2}\cancel{\left(V_1^2\right)} + \cancel{gh_1} = \frac{P_2}{\rho} + \frac{1}{2}\left(V_2^2\right) + \cancel{gh_2}$$

Show on board

$$\frac{P_1}{\rho} = \frac{P_2}{\rho} + \frac{1}{2}\left(V_2^2\right) \longrightarrow \frac{P_1}{\rho} - \frac{P_2}{\rho} = \frac{1}{2}\left(V_2^2\right)$$

$$\frac{P_1 - P_2}{\rho} = \frac{1}{2}\left(V_2^2\right) \quad or \quad V = \sqrt{\frac{2\Delta P}{\rho}}$$

What to look for in an Equation

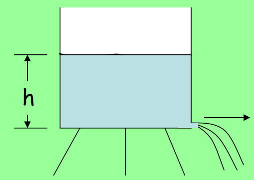
- Dimensions or Units ✓
- Principles ✓
- Assumptions
- Simple relationships ✓
- Simplifications ✓

Assumptions

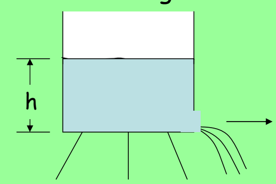
- incompressible fluid
- steady flow
- along a single flowing streamline (single small packet of fluid)
- no viscous (fluid frictional type) losses
- no work done on the fluid

Do these tanks drain at the same rate?

Tank with small exit



Tank with large exit



What does this equation show?

$$\overset{\sim 0}{\cancel{\frac{\Delta P}{\rho}}} + \frac{1}{2}\Delta\left(V^2\right) + g\Delta h = 0$$

What is missing?

What to look for in an Equation

- Dimensions or Units ✓
- Principles ✓
- Assumptions ✓
- Simple relationships ✓
- Simplifications ✓

Occam's Razor

Non sunt multiplicanda entia praeter necessitatem
Willima of Occam, 14th Century

"Things should not be multiplied without good reason"

Make everything as simple as possible, but not simpler – Einstein

Possible Exercise (if time is left)

- What would you need to know to design a Wind Chime?
- Sound – a review
- Compression waves (slinky exercises)
  - transverse vs longitudinal waves
  - mix in the air
- Pure tone would be a sine wave (tuning fork)
  - amplitude
  - frequency
- Tone is set by shape (M3)
  - wave shape (harmonics, type of wave)
  - envelope shape (attack, decay, sustain, release – vibrato . .)