

**Function Discovery – Two-parameter Models
for functions that are monotonic for positive x values**

Table 1: Basic Two-Parameter Models

Model type	Equation	Linearized form	Plot type (MATLAB command)	Fit
linear	$y = mx + b$	$y = mx + b$	linear (plot)	x vs y
exponential	$y = be^{mx}$	$\ln(y) = \ln(b) + mx$	semilog (semilogy)	x vs $\ln(y)$
	$y = b10^{mx}$	$\log(y) = \log(b) + mx$		x vs $\log_{10}(y)$
power	$y = bx^m$	$\log(y) = \log(b) + m \log(x)$	log-log (loglog)	$\log_{10}(x)$ vs $\log_{10}(y)$

Assessing the function type:

1. Plot the data you have,
2. For linear functions: If the graph forms a straight line, it is linear and you can go on to fitting the parameters for a simple linear model.
3. If it is not straight, is it monotonic for positive independent variable? If it is not monotonic than this approach will not work.
4. For non-linear monotonic functions: Prepare a log-log plot and a semilog plot – is it linear on either of these plots? If so you can proceed to fitting the parameters for the model associated with that plot:
 - linear on a semilog implies an exponential model,
 - linear on a log-log plot implies a power model.

Table 2: MATLAB commands of interest

Action	MATLAB Commands	comments
Plot of actual and predicted values on linear axes	<code>>> plot(x,y,'*',x,yhat);</code>	data as points, equation as line
Plot a log-log graph (both axes are logarithms)	<code>>> loglog(x,y)</code>	has same format and options as the plot command
Plot a semilog graph (with a linear x-axis and a log y-axis)	<code>>> semilogy(x,y)</code>	has same format and options as the plot command
calculate common (base 10) logarithms	<code>>> log10(x)</code>	
calculate natural (base e) logarithms	<code>>> log(x)</code>	

Some data to try (all data sets from Palm): For each data set below determine if the data is best fit by a linear, exponential or power law model – explain your reasoning.

1. Deflection of a cantilever beam

Force - F(lbs.)	0	100	200	300	400	500	600	700	800
Deflection - d(in.)	0	0.09	0.18	0.28	0.37	0.46	0.55	0.65	0.74

2. x1 vs y1

x1	2.5	3	3.5	4	4.5	5	5.5	6	7	8	9	10
y1	1500	1220	1050	915	810	745	690	620	520	480	410	390

3. x2 vs y2

x2	550	600	650	700	750
y2	41.2	18.62	8.62	3.92	1.86

Spring EG – Summary of MATLAB Commands for a determining a linear fit

Model: $\text{Length} = b + m * \text{Force}$

Setup: Starting with row vectors Length & Force:

```
>> X = [ones( length( Force ), 1), Force]; % Create the design matrix (x values must be a column)
>> Y = Length; % Set up response vector (must be a column)
```

Fit:

```
>> B = X \ Y % Calculate the parameter vector, [b m]'
B =
    0.0758
    4.9990
```

Predict:

```
>> Yhat = X*B; % Calculate the predicted values
```

Graph the result:

```
>> plot(X(:,2), Y, 'x', X(:,2), Yhat) % Plot actual and predicted values
>> xlabel 'Force(lbf)', ylabel 'Length(in.)' % Label axes
>> legend('Experimental Data', 'Fit: L = 5*F + 0.076') % Add legend with fitted equation
```