Vector/Matrix multiplication (section 2.4)

1. Vector multiplication (the inner or dot product)
   a. A row vector times a column vector (of the same length)
   b. Each term multiplied by the equivalent term in other vector and then the results are summed
   c. Example:

$$u \cdot v = \begin{bmatrix} 4 & 3 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 1 \end{bmatrix} = (4*5) + (3*1) = 23$$

   d. In general notation

$$u \cdot v = \begin{bmatrix} u_1 & u_2 & \ldots & u_n \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} = (u_1 v_1 + u_2 v_2 + \ldots + u_n v_n$$

   e. You try (by hand)

$$u \cdot v = \begin{bmatrix} 3 & 1 & 7 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 6 \\ 5 \end{bmatrix} = ?$$

2. Matrix multiplication
   a. Every row in the first matrix is vector multiplied by every column in the second matrix. The answer is put in the row column position defined by the row number from the first matrix and the column number of the second matrix.

   b. General form:

$$U * V = \begin{vmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \\ u_{31} & u_{32} \end{vmatrix} \cdot \begin{vmatrix} v_{11} & v_{12} & v_{13} \\ v_{21} & v_{22} & v_{23} \end{vmatrix} = \begin{vmatrix} u_{11}v_{11} + u_{12}v_{21} & u_{11}v_{12} + u_{12}v_{22} & \ldots \\ u_{21}v_{11} + u_{22}v_{21} & \ldots & \ldots \\ \ldots & \ldots & u_{31}v_{13} + u_{32}v_{23} \end{vmatrix}$$

   c. Example Problem

Matrix Mult.

$$\begin{vmatrix} 6 & -2 \\ 10 & 3 \\ 4 & 7 \end{vmatrix} \cdot \begin{vmatrix} 9 & 8 \\ -5 & 12 \end{vmatrix} = \begin{vmatrix} (6)(9) + (-2)(-5) = 64 & 24 \\ (10)(9) + (3)(-5) = 75 & 116 \\ 1 & 116 \end{vmatrix}$$

   d. Pattern of dimensions: [m x n][n x p] = [m x p]

   e. You try

$$\begin{vmatrix} 4 & 3 \\ -1 & 6 \end{vmatrix} \cdot \begin{vmatrix} 5 & 1 \\ 3 & 7 \end{vmatrix} = ?$$

Practice Problems (calculate in MATLAB, you should also be able to do by hand):

1. $\begin{bmatrix}8 & 2 & 1\end{bmatrix}\begin{bmatrix}3\\5\\11\end{bmatrix} = ?$

2. $A = \begin{bmatrix}11 & 5\\-9 & -4\end{bmatrix} \qquad B = \begin{bmatrix}-7 & -8\\6 & 2\end{bmatrix}$

   a.) A*B = ?

   b.) B*A = ?

   c.) A.*B=?

3. $M = \begin{bmatrix}3 & 4 & 5\\-5 & 13 & 4\end{bmatrix} \qquad N = \begin{bmatrix}6 & 1\\-4 & 7\\6 & 2\end{bmatrix}$

   a.) M*N = ?

   b.) N*M, = ?

   c.) what happens when you try N.*M, why?

Solution - Practice Problems

1. $\begin{bmatrix}8 & 2 & 1\end{bmatrix}\begin{bmatrix}3\\5\\11\end{bmatrix} = ?$

```
>> [8 2 1]*[3 5 11]'
ans =
   45
```

2. $A = \begin{bmatrix}11 & 5\\-9 & -4\end{bmatrix} \qquad B = \begin{bmatrix}-7 & -8\\6 & 2\end{bmatrix}$

a.) A*B = ?

b.) B*A = ?

c.) A.*B=?

```
>> A=[11 5; -9 -4];B=[-7 -8; 6 2];
>> A*B
ans =
  -47  -78
   39   64
>> B*A
ans =
   -5   -3
   48   22
>> A.*B
ans =
  -77  -40
  -54   -8
```

3. $M = \begin{bmatrix}3 & 4 & 5\\-5 & 13 & 4\end{bmatrix} \qquad N = \begin{bmatrix}6 & 1\\-4 & 7\\6 & 2\end{bmatrix}$

a.) M*N = ?

b.) N*M = ?

d.) what happens when you try N.*M, why?

```
>> M=[3 4 5;-5 13 4];
>> N=[6 1; -4 7;6 2];
>>A = M*N
A =
   32   41
  -58   94
>> B = N*M
B =
   13   37   34
  -47   75    8
    8   50   38
>> C = N.*M
??? Error using ==> times
Matrix dimensions must agree.
```

In this case you get an error because N & M have different dimensions